

ADA 016630

JASON

Technical Report JSR-74-6

June 1975

LOW-FREQUENCY SOUND PROPAGATION IN A FLUCTUATING INFINITE OCEAN--II

By: F. ZACHARIASEN

Contract No. DAHC15-73-C-0370 ✓
ARPA Order No. 2504
Program Code No. 3K10
Date of Contract: 2 April 1973
Contract Expiration Date: 30 June 1975
Amount of Contract: \$1,297,321

Approved for public release; distribution unlimited.



Sponsored by

DEFENSE ADVANCED RESEARCH PROJECTS AGENCY
ARPA ORDER NO. 2504



STANFORD RESEARCH INSTITUTE
Menlo Park, California 94025 • U.S.A.

Copy No. 218

The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the Defense Advanced Research Projects Agency or the U.S. Government.

Related Rept - AD787041

ADDITIONAL BY		
BY	DATE	<input checked="" type="checkbox"/>
DATE	DATE	<input type="checkbox"/>
UNAPPROVED		<input type="checkbox"/>
JUSTIFICATION		
BY		
DISTRIBUTION/AVAILABILITY CODES		
DIAL	ATIL	SPECIAL
A		

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM	
1. TITLE (AND Subtitle) JSR-74-6 (14) SRI		2. GOVT ACCESSION NO.	
3. RECIPIENT'S CATALOG NUMBER		5. TYPE OF REPORT & PERIOD COVERED Technical Report	
4. AUTHOR(s) F. Zachariassen		6. REPORT NUMBER SRI 3000	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Stanford Research Institute Menlo Park, California 94025		8. CONTRACT OR GRANT NUMBER(s) DAHC15-73-C-0379 ARPA Order - 25041	
11. CONTROLLING OFFICE NAME AND ADDRESS Defense Advanced Research Projects Agency 1400 Wilson Boulevard Arlington, Virginia 22209		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS (12) 38p.	
14. MONITORING AGENCY NAME & ADDRESS (if diff. from Controlling Office)		12. REPORT DATE June 1975	
		13. NO. OF PAGES 36	
		15. SECURITY CLASS. (of this report) Unclassified	
16. DISTRIBUTION STATEMENT (of this report) Approved for public release; distribution unlimited		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from report)			
18. SUPPLEMENTARY NOTES			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Internal waves Sound propagation fluctuations			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) An approximation method for calculating fluctuations due to internal waves in sound propagation in the ocean is outlined, including the effects of the sound channel. Results are presented in a simple form in which the geometrical optics limit is transparent.			

DD FORM 1473
1 JAN 73
EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

332500

4/8

CONTENTS

DD Form 1473

I	INTRODUCTION.	1
II	REVIEW OF SUPEREIKONAL APPROXIMATION.	2
III	FLUCTUATIONS IN A HOMOGENEOUS ISOTROPIC OCEAN	9
IV	RYTOV'S METHOD IN AN INHOMOGENEOUS OCEAN.	15
V	FLUCTUATIONS IN AN INHOMOGENEOUS OCEAN.	20
VI	CONCLUSION.	26
	REFERENCES.	27

I INTRODUCTION

In an earlier report,^{1*} we described an approximation scheme, called the "supereikonal approximation," for calculating fluctuations produced by a specified spectrum of sound-speed fluctuations, in pressure signals from a CW source in the ocean. In that report, our analysis was confined to the case where the fluctuations were superimposed on a homogeneous isotropic ocean.

Here we should like to extend and improve on the earlier version in two major ways. First, we should like to rewrite the results obtained in Ref. 1 for the homogeneous case in a much simpler form, in which the geometrical optics limit valid for short ranges is transparent, and in which the corrections to this limit that became important at large ranges are easily visualized. We should also like to derive formulas for the fluctuations in the quantities of most immediate experimental interest--namely, the mean square phase of the received pressure, and the mean square logarithm of the amplitude of the received pressure.

Second, we should like to extend our treatment to the realistic case in which the background ocean is not isotropic and homogeneous, but contains a sound channel. As we shall see, we can handle this situation as well, with only a modest increase in complexity, as long as the fluctuations and ranges are such that separate ray paths are still identifiable. Our result will then apply to fluctuations in the signal associated with a given ray path.

Numerical results, and specific models of fluctuation spectra, will be treated in a separate report.

*References are listed at the end of the report.

II REVIEW OF SUPEREIKONAL APPROXIMATION

Let us begin our discussion by neglecting the effects of the sound channel. The problem of sound propagation in the presence of fluctuations superimposed on a homogeneous isotropic background is easier to set up and to visualize than when the fluctuations are superimposed on an inhomogeneous background, so it is conceptually advantageous to work out this case first. Then, as we shall see later, when the inhomogeneous background representing the sound channel is introduced, the analysis can be carried out very much as in the homogeneous case, and the resulting formulas, while geometrically more complex, are entirely analogous to those obtained in the simpler example.

Our analysis will be based on the supereikonal approximation, and it will be convenient at this point to give a brief review of the results of this method. The problem is to evaluate the pressure $p(\vec{x})$ at a point \vec{x} produced by a point source (which, for convenience, we can take to have unit strength) located at the origin. The sound propagation from the source to the point \vec{x} is through an isotropic homogeneous ocean, in which the sound speed is c , on which is superimposed a fluctuation in sound speed $\delta c(\vec{x})$ that is, of course, very small compared to c . Mathematically, then, the pressure satisfies the wave equation

$$(\nabla^2 + k^2)p(\vec{x}) = v(\vec{x})p(\vec{x}) \quad (2.1)$$

where $k = \omega/c$ for a source emitting sound of frequency ω , and where

$$v(\vec{x}) = 2k^2 \delta c(\vec{x})/c \quad (2.2)$$

The boundary condition associated with Eq. (2.1) is that as $\vec{x} \rightarrow 0$,

$$p(\vec{x}) \rightarrow \frac{1}{4\pi x} \quad (2.3)$$

(we shall denote $|\vec{x}|$ simply by x)

(We may comment that in the case of an inhomogeneous background, in Eq. (2.1), k is simply replaced by $k(\vec{x}) = \omega/c(\vec{x})$, where $c(\vec{x})$ is the sound speed in the inhomogeneous background.)

Equation (2.1) may be cast into integral form through the use of the outgoing-wave Green's function

$$(\nabla^2 + k^2)\Delta(\vec{x} - \vec{y}) = \delta^3(\vec{x} - \vec{y}) \quad ; \quad (2.4)$$

explicitly, we have

$$\Delta(\vec{x}) = \frac{1}{4\pi x} e^{ikx} \quad (2.5)$$

Then we may write, in place of Eq. (2.1),

$$p(\vec{x}) = \Delta(\vec{x}) + \int d^3y \Delta(\vec{x} - \vec{y}) V(\vec{y}) p(\vec{y}) \quad (2.6)$$

Iteration of this integral equation generates the perturbation series for $p(\vec{x})$, which is more convenient to write in Fourier-transformed form, as follows:

$$\begin{aligned} p(\vec{q}) = & \Delta(\vec{q}) + \Delta(\vec{q}) \int \frac{d^3\vec{q}_1}{(2\pi)^2} V(\vec{q}_1) \Delta(\vec{q} - \vec{q}_1) \\ & + \Delta(\vec{q}) \int \frac{d^3\vec{q}_1}{(2\pi)^3} \int \frac{d^3\vec{q}_2}{(2\pi)^3} V(\vec{q}_1) \Delta(\vec{q} - \vec{q}_1) V(\vec{q}_2) \Delta(\vec{q} - \vec{q}_1 - \vec{q}_2) \\ & + \dots \end{aligned} \quad (2.7)$$

where

$$\Delta(\vec{q}) = \frac{1}{q^2 - k^2 + i\epsilon}, \quad (2.8)$$

and, of course, where

$$p(\vec{q}) = \int d^3\vec{x} e^{-i\vec{q}\cdot\vec{x}} p(\vec{x}). \quad (2.9)$$

The supereikonal approximation now consists of neglecting all momentum transfer correlations in the perturbation series. That is, we approximate $(q - q_1 - q_2 - \dots - q_n)^2 - k^2 + i\epsilon$ by $q^2 - 2q \cdot (q_1 + q_2 + \dots + q_n)^2 + q_1^2 + q_2^2 + \dots + q_n^2 - k^2 + i\epsilon$, and neglect all terms of the form $\vec{q}_i \cdot \vec{q}_j$ when $i \neq j$. Note that the first approximation occurs in the second-order term in V . Once this simplification is made, the perturbation series can be summed exactly, and one obtains the result

$$p(\vec{x}) = \frac{\sqrt{-i\pi}}{8\pi^2} \int_0^\infty \frac{d\beta}{\beta^{3/2}} e^{i[\beta k^2 + \frac{x^2}{4\beta} + \beta I(\beta, \vec{x}) + i\epsilon]} \quad (2.10)$$

where

$$I(\beta, \vec{x}) = \int \frac{d^3\vec{q}}{(2\pi)^3} V(\vec{q}) \int_0^1 ds e^{-i[sq \cdot x + \beta s(1-s)q^2]} \quad (2.11)$$

This expression constitutes the supereikonal approximation to the pressure. The conditions under which it is valid are

$$k x \gg 1, \quad k L \gg 1$$

and

$$x \ll \frac{1}{k^2 L} \left(\frac{c}{\delta c} \right)^2. \quad (2.12)$$

(In fact, the last condition may well be too stringent.) Here L is the correlation length of the sound-speed fluctuations--i.e., the correlation function $C(\vec{x} - \vec{y}) \equiv \langle V(\vec{x}) V(\vec{y}) \rangle$ vanishes when $|\vec{x} - \vec{y}| \gtrsim L$.

It is worth noting that if, in Eq. (2.11), the $\theta s(1-s)q^2$ term is omitted from the exponent, we obtain

$$p(\vec{x}) = \frac{1}{4\pi x} c^{i \left(\sqrt{k^2 + \int_0^1 V(s\vec{x}) ds} \right) x}$$

which is the conventional WKB, or eikonal, or geometrical optics, approximation to the pressure. The primary virtue of the supereikonal form, therefore, is that it contains as limiting cases both the conventional eikonal and complete first-order perturbation-theory approximations.

While Eqs. (2.10) and (2.11) do constitute a closed-form solution for the pressure, the expressions are still a bit unwieldy, and further simplification of them is useful. To this end, let us evaluate the integral in Eq. (2.10) by stationary phase, keeping in mind that x and k are both large. The stationary phase point is β_0 , where

$$k^2 - \frac{x^2}{4\beta_0^2} + I(\beta_0, \vec{x}) + \beta_0 \frac{\partial}{\partial \beta} I(\beta, \vec{x}) \Big|_{\beta=\beta_0} - 3/2\beta_0 = 0.$$

From Eq. (2.11), we may estimate that

$$\beta_0 \frac{\partial}{\partial \beta} I(\beta, \vec{x}) \Big|_{\beta=\beta_0} \sim \frac{k^2}{L^2} \cdot \frac{\delta c}{c} \cdot \beta_0 ;$$

hence, if

$$x < k L^2 \frac{c}{\delta c} , \quad (2.13)$$

the stationary phase point is accurately given by the solution of the simpler equation

$$k^2 - x^2/4\beta_0^2 = 0$$

and is located at $\beta_0 = x/2k$. Thus we find

$$p(x) = \frac{e^{ikx}}{4\pi x} e^{\frac{ix}{2k} I\left(\frac{x}{2k}, \vec{x}\right)} . \quad (2.14)$$

To approximately evaluate the integral I in this expression, we recall that the supereikonal approximation should be exact to first order in V . This requirement then yields the result

$$p(\vec{x}) = \Delta(\vec{x}) \exp \frac{1}{\Delta(\vec{x})} \int d^3\vec{y} \Delta(\vec{x}-\vec{y}) V(\vec{y}) \Delta(\vec{y}) . \quad (2.15)$$

This expression is known as Rytov's approximation to the pressure. A direct derivation of it may be made by replacing the wave equation (2.1) by an equation for $\log[p(\vec{x})/\Delta(\vec{x})]$ and solving this to first order in V .² However the justification for the approximation is somewhat obscure in

this direct derivation; in the approach via the supereikonal technique, what is being left out is more clearly visualized.

In any event, depending on the validity of the criterion (2.13), one may use either the supereikonal expression (2.10) or the Rytov expression (2.15) to proceed further.* We shall use (2.15).

Let us write, then,

$$p(\vec{x}) = \Delta(\vec{x}) e^{X(\vec{x})} \quad (2.16)$$

where

$$X(\vec{x}) = \frac{1}{\Delta(\vec{x})} \int d^3\vec{x}' \Delta(\vec{x}-\vec{x}') V(\vec{x}') \Delta(\vec{x}') \quad (2.17)$$

For our homogeneous background case, where the Green's function Δ is given by Eq. (2.5), we have

$$X(\vec{x}) = \frac{1}{4\pi} \int d^3\vec{x}' \frac{x}{x' |\vec{x}-\vec{x}'|} e^{ik(\vec{x}' + |\vec{x}-\vec{x}'| - x)} V(\vec{x}') \quad (2.18)$$

In concluding this section, and as an aside, let us comment on the geometrical optics limit of this expression. This limit results from an evaluation of $X(\vec{x})$ by the method of stationary phase. Provided that the Fresnel condition

$$x < k L^2$$

*

All of these statements, we remind the reader, are subject to the range constraints under which the supereikonal expression was derived in the first place.

is met, the stationary phase path in Eq. (2.18) is the straight line joining 0 to \vec{x} , and the stationary phase value of $X(\vec{x})$ is just

$$X(x,0,0) = \frac{i}{2k} \int_0^x dx' V(x',0,0) \quad , \quad (2.19)$$

which is immediately recognized as the correct geometrical optics expression for the phase. The analogous expression for the amplitude in geometrical optics is obtained by keeping the second-order transverse derivatives in V as well.¹

III FLUCTUATIONS IN A HOMOGENEOUS ISOTROPIC OCEAN

The quantities that it will be of interest to compute are the statistical averages $\langle X^2(\vec{x}) \rangle$ and $\langle |X(\vec{x})|^2 \rangle$. (We note that $\langle X(\vec{x}) \rangle = 0$, of course.) These are connected to phase and amplitude fluctuations $\langle \phi^2 \rangle$ and $\langle |\log p/p_0|^2 \rangle \equiv \langle A^2 \rangle$ by the relations

$$\langle \phi^2 \rangle = 1/2 \left(\langle |X|^2 \rangle - \text{Re} \langle X^2 \rangle \right) \quad (3.1)$$

and

$$\langle A^2 \rangle = 1/2 \left(\langle |X|^2 \rangle + \text{Re} \langle X^2 \rangle \right) \quad (3.2)$$

We will, in addition, obtain the cross correlation $\text{Im} \langle X^2 \rangle$.

Insofar as the sound-speed fluctuations, and hence the fluctuations in X , are gaussian, the pressure fluctuations are related to the statistical averages as well. We have

$$\langle p \rangle = p_0 e^{1/2 \langle X^2 \rangle}$$

$$\langle p^2 \rangle = p_0^2 e^{2 \langle X^2 \rangle}$$

$$\langle |p|^2 \rangle = |p_0|^2 e^{1/2 \langle (X+X^*)^2 \rangle}$$

where we write

$$p_0(\vec{x}) \equiv \Delta(\vec{x}) = \frac{e^{ikx}}{4\pi x}$$

as the received pressure from the unit point source in the absence of fluctuations.

Eventually, we will also be interested in correlations; these will involve averages such as $\langle X(\vec{x}_1)X(\vec{x}_2) \rangle$, etc., but we shall ignore these for now.

Let us first evaluate $\langle |X(\vec{x})|^2 \rangle$. For convenience, we shall choose \vec{x} to lie along the x axis, so that $\vec{x} = (x, 0, 0)$. From the definition, Eq. (2.18), we have

$$\begin{aligned} \langle |X(\vec{x})|^2 \rangle &= \left(\frac{1}{4\pi} \right)^2 \int d^3\vec{y}_1 \int d^3\vec{y}_2 \frac{x^2}{y_1 |x - \vec{y}_1| y_2 |x - \vec{y}_2|} \\ &\quad e^{ik(y_1 + |x - \vec{y}_1| - x)} e^{-ik(y_2 + |x - \vec{y}_2| - x)} \\ &\quad c(\vec{y}_1 - \vec{y}_2) \end{aligned} \quad (3.3)$$

where we have introduced the correlation function

$$c(\vec{y}_1 - \vec{y}_2) \equiv \langle v(\vec{y}_1) v(\vec{y}_2) \rangle .$$

We assume C to be independent of $(y_1 + y_2)/2$ for this case of a homogeneous background.

It is convenient in Eq. (3.3) to shift to relative and center-of-mass coordinates. We define

$$\vec{y} = y_1 - y_2 , \quad \vec{Y} = \frac{y_1 + y_2}{2} .$$

Then, if we assume that $C(\vec{y})$ cuts off for values of $y \geq L$ where $L \ll x$, we may expand in y/Y . Thus, Eq. (3.3) becomes

$$\langle |X(\vec{x})|^2 \rangle = \left(\frac{1}{4\pi} \right)^2 \int d^3\vec{Y} \frac{x^2}{Y^2 |Y - x|^2} \int d^3\vec{y} C(\vec{y}) \exp ik \vec{y} \cdot [\hat{Y} - (x \wedge Y)] \quad (3.4)$$

Here \hat{Y} and $x \wedge Y$ stand for unit vectors in the direction of \vec{Y} and $x \wedge Y$, respectively, and we have written $|Y \pm y/2| \approx Y$, $|x - Y \pm y/2| \approx |x \wedge Y|$ in the geometrical factors multiplying the exponentials. This approximation introduces a negligible error.

The integral over $d^3\vec{Y}$ may now be evaluated by stationary phase. The stationary phase path is the straight line joining 0 to \vec{x} , and the result is

$$\langle |X(\vec{x})|^2 \rangle = \frac{x}{4k} \int dy_{||} C(y_{||}, 0) \quad (3.5)$$

where $y_{||}$ refers to the component of \vec{y} in the direction parallel to \vec{x} .

Introducing the Fourier transform of the correlation function

$$\tilde{C}(\vec{q}) = \int d^3\vec{r} e^{-i\vec{q} \cdot \vec{r}} C(\vec{r}) \quad (3.6)$$

permits us to rewrite Eq. (3.5) in the sometimes more convenient form

$$\langle |X(\vec{x})|^2 \rangle = \left(\frac{1}{4\pi k} \right)^2 x \int d^2\vec{q}_{\perp} \tilde{C}(0, \vec{q}_{\perp}) \quad (3.7)$$

where " \perp " refers to the directions perpendicular to \vec{x} .

Next let us turn to $\langle X^2 \rangle$. We now have, instead of Eq. (3.3), the expression

$$\begin{aligned} \langle X(\vec{x})^2 \rangle &= \left(\frac{1}{4\pi} \right)^2 \int d^3 \vec{y}_1 \int d^3 \vec{y}_2 \frac{x^2}{y_1 |\vec{x} - \vec{y}_1| y_2 |\vec{x} - \vec{y}_2|} \\ &\quad e^{ik(y_1 + |\vec{x} - \vec{y}_1| - x)} e^{ik(y_2 + |\vec{x} - \vec{y}_2| - x)} \\ &\quad C(y_1 - y_2) \end{aligned} \quad (3.8)$$

We again shift to the variables \vec{Y} and \vec{y} , and appeal to the vanishing of $C(\vec{y})$ for $y \gtrsim L$ to justify expanding in y/Y and $y/|\vec{x} - \vec{Y}|$. We obtain

$$\begin{aligned} \langle X(\vec{x})^2 \rangle &= \left(\frac{1}{4\pi} \right)^2 \int d^3 \vec{Y} \frac{x^2}{Y^2 |\vec{x} - \vec{Y}|^2} e^{2ik(Y + |\vec{x} - \vec{Y}| - x)} \int d^3 \vec{y} C(\vec{y}) \\ &\quad \exp \frac{ik}{4} \left(\frac{y^2 - (\vec{y} \cdot \hat{Y})^2}{Y} + \frac{y^2 - [\vec{y} \cdot (\vec{x} - \vec{Y})]^2}{|\vec{x} - \vec{Y}|} \right). \end{aligned} \quad (3.9)$$

As before, we may evaluate the integral over $d^3 Y$ by stationary phase. This yields

$$\begin{aligned} \langle X(\vec{x})^2 \rangle &= \left(\frac{1}{4\pi} \right)^2 \int_0^x ds \int dy_{||} \int d^2 \vec{y}_{\perp} C(y_{||}, y_{\perp}) \\ &\quad \frac{x^2}{s^2 (x-s)^2} e^{\frac{ik}{4} \left(\frac{1}{s} + \frac{1}{x-s} \right) y_{\perp}^2} \end{aligned} \quad (3.10)$$

where, again, " $||$ " and " \perp " refer to directions parallel and perpendicular to \vec{x} .

At this point it is convenient to express $C(y_{\parallel}, \vec{y}_{\perp})$ in terms of its Fourier transform, as given by Eq. (3.6). The integral over $dy_{\parallel} d^2y_{\perp}$ can then be carried out, and we finally obtain the relatively simple expression

$$\tilde{C} \langle X(\vec{x})^2 \rangle = - \left(\frac{1}{4\pi k} \right)^2 \int d^2\vec{q}_{\perp} \tilde{C}(0, \vec{q}_{\perp}) \int_0^x ds e^{i \frac{q_{\perp}^2}{k} \frac{(s-x)s}{x}} \quad (3.11)$$

Equations (3.7) and 3.11) constitute our central results. They express the quantities of interest as integrals along unperturbed ray paths (in this case straight lines) of the Fourier transform of the correlation function $C(\vec{q})$ times rather simple geometrical factors. As we shall see later, entirely parallel expressions obtain in the more difficult case of an inhomogeneous background medium.

The expression for $\langle |X(\vec{x})|^2 \rangle$, Eq. (3.7), is precisely the same result for this quantity obtained by using geometrical optics to compute $X(\vec{x})$ itself, and then calculating $\langle |X(\vec{x})|^2 \rangle$ from this. [This is easily seen by referring back to Eq. (2.19).] In contrast, Eq. (3.11) is not what one obtains for $\langle X(\vec{x})^2 \rangle$ from geometrical optics. Geometrical optics for this quantity is recovered if one expands the exponential in Eq. (3.11), a procedure that evidently is valid only if

$$\frac{q_{\perp}^2}{k} \frac{(s-x)s}{x} \ll 1$$

Since $q_{\perp} \sim 1/L$ and $s, x-s \sim x$, this condition can be more familiarly written as

$$x \ll k L^2$$

which we recognize as the Fresnel condition under which the geometrical optics approximation for $X(\vec{x})$ itself was valid in the first place.

Thus, Eq. (3.11) constitutes an improvement over geometrical optics, while Eq. (3.7) coincides with geometrical optics. Conversely, geometrical optics for $\langle |X|^2 \rangle$ is valid out to a very large range, while geometrical optics for $\langle X^2 \rangle$ is valid only within the range $x < k L^2$.

It is of interest to study Eq. (3.11) in the limit of very long range. As $x \rightarrow \infty$, the integral over ds can be approximately evaluated, and we find¹

$$\langle X^2(\vec{x}) \rangle \approx \frac{iC(0)}{8\pi k} (\gamma + \log 4kx - i\pi/2) \quad (3.12)$$

where $\gamma = 0.577\dots$ is Euler's constant. Thus, for small x satisfying the Fresnel condition, we have

$$\langle X^2(\vec{x}) \rangle \approx - \left(\frac{1}{4\pi k} \right)^2 \int d^2 q_{\perp} \tilde{C}(0, \vec{q}_{\perp}) \left[x + \frac{i q_{\perp}^2}{6k} x^2 + \dots \right], \quad (3.13)$$

and for very large x we have $\langle X^2(\vec{x}) \rangle \sim i \log x$ as given by Eq. (3.12). Between these two extremes, of geometrical optics and of very long ranges, Eq. (3.11) provides a smooth interpolation.

Equation (3.7), on the other hand, gives $\langle |X|^2 \rangle$ for all values of x , large and small, and simply says that $\langle |X|^2 \rangle$ is proportional to the range x everywhere.

IV RYTOV'S METHOD IN AN INHOMOGENEOUS OCEAN

Now let us turn to the effects of the sound channel. That is, we must replace the nonfluctuating sound speed c in the homogeneous case by a (specified) function of position $c(\vec{x})$. In fact, for the ocean, $c(\vec{x}) = c(z)$ is a function of depth only. A reasonably explicit form for $c(z)$ is the relatively simple expression³

$$c(z) = c_A [1 + \epsilon(e^{-\eta} + \eta - 1)]$$

where $\eta = (z - z_A)/\frac{1}{2}B$, where z_A is the sound-channel depth, and ϵ and B are parameters. We shall, however, write our formulas for a general $c(\vec{x})$ until the time comes to make explicit numerical estimates.

The wave equation for the pressure, which is our starting point, now becomes altered from Eq. (2.1) to the equation

$$[\nabla^2 + k^2(\vec{x})]p(\vec{x}) = V(\vec{x})p(\vec{x}) \quad , \quad (4.1)$$

still with the same boundary condition that

$$p(\vec{x}) \rightarrow \frac{1}{4\pi x}$$

as $\vec{x} \rightarrow 0$, corresponding to an isotropic point source at the origin, where now $k(\vec{x}) = \omega/c(\vec{x})$.

We must first study the nonfluctuating part of the problem, to evaluate the Green's function in the presence of the sound channel. This satisfies

$$[\nabla^2 + k^2(\vec{x})]\Delta(\vec{x},\vec{y}) = \delta^3(\vec{x}-\vec{y}) \quad (4.2)$$

Note that it is no longer a function only of $\vec{x}-\vec{y}$ as it was in the homogeneous case. We shall assume that geometrical optics provides a good approximation to the nonfluctuating sound-channel problem. This means that we can represent $\Delta(\vec{x},\vec{y})$ as a sum of contributions from each ray joining \vec{x} and \vec{y} . To be specific, we may write

$$\Delta(\vec{x},\vec{y}) = \sum_{i=1}^{n(\vec{x},\vec{y})} \Delta_i(\vec{x},\vec{y}) \quad (4.3)$$

where $n(\vec{x},\vec{y})$ is the number of rays and Δ_i is the contribution of the i^{th} ray. We have, in particular for rays joining the origin and \vec{x} ,

$$\Delta_i(\vec{x},0) = K_i(\vec{x},0) \exp i \int_0^{\vec{x}} ds k(\vec{x}_i(s)) \quad , \quad i = 1 \dots n(\vec{x}), \quad (4.4)$$

where ds is an element of path length along the ray, $\vec{x}_i(s)$ is the i^{th} ray joining 0 to \vec{x} , and K_i is a normalization factor.

Now when the fluctuations are turned on, the signals traveling on each of the rays joining the origin to the point of observation \vec{x} are subject to small-angle scatterings by the perturbing potential $V(\vec{x})$. The signals are thus deflected slightly from the undisturbed rays by each interaction with V . The repeated action of V thus produces, on each ray, a sort of random walk of the signal away from the original ray. When we average over an ensemble of perturbations V , the disturbed signals will fill up a tube surrounding the undisturbed ray. Provided that these tubes

around each of the original rays do not overlap, the received pressure will be a sum of contributions from each ray tube.

We may estimate the radius of a ray tube as follows. The mean free path d between interactions of the signal traveling along a given ray with the perturbing potential V is of the order of $kc/\delta c$. Hence, over a range x the number of scatterings is $n = x/d$. The average deflection angle due to each scattering is of the order of $1/kL_V$ vertically and $1/kL_H$ horizontally, where L_V and L_H are the vertical and horizontal correlation lengths of the sound-speed fluctuations. Since the process is a random walk, the net displacement due to n collisions is proportional to \sqrt{n} , and hence the vertical and horizontal extents of the tube are, roughly,

$$r_V \sim \sqrt{\frac{x}{k}} \frac{1}{kL_V} \sqrt{\frac{c}{\delta c}}$$

and

$$r_H \sim \sqrt{\frac{x}{k}} \frac{1}{kL_H} \sqrt{\frac{c}{\delta c}} .$$

Let us assume that the vertical extent of the tubes is small enough so that the tubes remain distinct. Then the pressure at \vec{x} is the sum of contributions from each tube,

$$p(\vec{x}) = \sum_{i=1}^{n(\vec{x})} p_i(\vec{x}) , \quad (4.5)$$

where $n(\vec{x})$ is the number of unperturbed rays joining the source to the point \vec{x} . We shall be interested in $p_i(\vec{x})$.

We note that $p_i(\vec{x})$ is the pressure that would be received at \vec{x} if the source were not isotropic, but rather emitted all its energy in the direction of the i^{th} unperturbed ray. Thus $p_i(\vec{x})$ must satisfy the wave equation (4.1) but with an anisotropic boundary condition that itself depends on \vec{x} . To make this more precise, let us define $p_i(\vec{y};\vec{x})$ to be the pressure at \vec{y} from a source at the origin that emits only within a small solid angle* around the direction of the i^{th} unperturbed ray joining the origin to \vec{x} . Thus $p_i(\vec{x}) = p_i(\vec{x};\vec{x})$, and furthermore $p_i(\vec{y},\vec{x})$ vanishes unless y is inside the i^{th} ray tube. Then

$$[\nabla_{\vec{y}}^2 + k^2(\vec{y})]p_i(\vec{y};\vec{x}) = V(\vec{y})p_i(\vec{y};\vec{x}), \quad i = 1 \dots n(\vec{x}) \quad (4.6)$$

In analogy with this definition of $p_i(\vec{y};\vec{x})$, we may also define an "unperturbed" Green's function $\Delta_i(\vec{y};\vec{x},0)$, $i = 1 \dots n(\vec{x})$, to satisfy

$$[\nabla_{\vec{y}}^2 + k^2(\vec{y})]\Delta_i(\vec{y};\vec{x},0) = 0, \quad (4.7)$$

again with the same boundary condition. This function, also, will vanish except when \vec{y} is near the i^{th} unperturbed ray.

We may now directly derive the analogue of Eq. (2.15) by computing the quantity $\log p_i(\vec{y};\vec{x})/\Delta_i(\vec{y};\vec{x},0)$ in perturbation theory, and using Eq. (4.2). We find, setting $\vec{y} = \vec{x}$, that

$$p_i(\vec{x}) = \Delta_i(\vec{x};\vec{x},0) e^{\frac{1}{\Delta_i(\vec{x};\vec{x},0)} \int_{i^{\text{th}} \text{ ray tube}} d^3\vec{x}' \Delta(\vec{x},\vec{x}') V(\vec{x}') \Delta(\vec{x}',0)} \quad (4.9)$$

We have here replaced $\Delta_i(\vec{x};\vec{x},0)$ simply by $\Delta_i(\vec{x},0)$. Equation (4.9) is evidently the generalization of the Rytov formula (2.15) to the situation of an inhomogeneous background and many rays. The expression clearly

fails if the range is so large that the ray tubes overlap; otherwise the validity conditions are the same as those in the homogeneous-background case.

V FLUCTUATIONS IN AN INHOMOGENEOUS OCEAN

In this section we shall use Eq. (4.9) to calculate the various averages of interest for the contribution of a single ray tube to the pressure in the presence of the sound channel. We shall, for simplicity, drop the index i , though we should keep in mind that when there are several unperturbed raypaths their contributions are to be added to obtain the total pressure. Our interest, then will be in the statistical fluctuations of the contributions of a single ray, or rather a single ray tube.

As in the homogeneous case, we define

$$X(\vec{x}) = \frac{1}{\Delta(\vec{x}, 0)} \int d^3\vec{y} \Delta(\vec{x}, \vec{y}) V(\vec{y}) \Delta(\vec{y}, 0) \quad (5.1)$$

and we wish to compute $\langle X^2 \rangle$ and $\langle |X|^2 \rangle$. We recall, from Section IV, that assuming geometrical optics to be a valid approximation for the nonfluctuating background permits us to write the Green's function as

$$\Delta(\vec{x}, \vec{y}) = K(\vec{x}, \vec{y}) e^{ikS(\vec{x}, \vec{y})} \quad (5.2)$$

where

$$kS(\vec{x}, \vec{y}) = \int_{\vec{x}}^{\vec{y}} ds \, k(\vec{x}(s)) \quad (5.3)$$

and where the normalization factor is

$$K(\vec{x}, \vec{y}) = \frac{1}{4\pi} \sqrt{\det \frac{\partial}{\partial x_{\perp i}} \frac{\partial}{\partial y_{\perp j}} S(\vec{x}, \vec{y})} \Big|_{\vec{x}_{\perp} = \vec{y}_{\perp} = 0}.$$

Here, \perp refers to directions perpendicular to the ray. (An excellent approximation to this, for our purposes, is to write simply $K(\vec{x}, \vec{y}) = (4\pi|\vec{x}-\vec{y}|)^{-1}$, as in the homogeneous-ocean case; we need to be careful about deviations from homogeneity only in the phases.) In Eq. (5.3) the line integral is along the ray of interest joining the points \vec{x} and \vec{y} .

To repeat our earlier calculations requires us to introduce the correlation function $\langle V(\vec{y}_1) V(\vec{y}_2) \rangle$. In the homogeneous case, this quantity depended only on the separation $y_1 - y_2$. Now, however, because of the background inhomogeneities, it will also depend on $(y_1 + y_2)/2$ (actually it will depend only on the mean depth $(z_1 + z_2)/2$ since the inhomogeneities depend only on depth). Thus we must now define the correlation function by

$$V(\vec{y}_1, \vec{y}_2) \equiv c\left(y_1 - y_2, \frac{y_1 + y_2}{2}\right).$$

As before, let us look first at $\langle |X|^2 \rangle$. We have

$$\begin{aligned} \langle |X(x)|^2 \rangle &= \int d^3 \vec{Y} \frac{K(\vec{x}, \vec{Y}) K(\vec{Y}, 0)}{K(\vec{x}, 0)} \int d^2 \vec{y} c(\vec{y}, \vec{Y}) \\ &\quad \exp i k [S(\vec{x}, \vec{Y} + \vec{y}/2) - S(\vec{x}, \vec{Y} - \vec{y}/2) \\ &\quad + S(\vec{Y} + \vec{y}/2, 0) - S(\vec{Y} - \vec{y}/2, 0)] \end{aligned} \quad (5.4)$$

and we must keep in mind that we are to integrate only over the ray tube surrounding the unperturbed ray of interest. In the homogeneous-background case we expanded the exponent in powers of \vec{y} , because $C(\vec{y})$ vanished for large $|\vec{y}|$. We may do the same here. Thus,

$$\langle |X(\vec{x})|^2 \rangle = \int d^3\vec{Y} \frac{K(\vec{x}, \vec{Y})K(\vec{Y}, 0)}{K(\vec{x}, 0)} \int d^3\vec{y} C(\vec{y}, \vec{Y}) \exp i k \vec{y} \cdot \vec{\nabla}_Y [S(\vec{x}, \vec{Y}) + S(\vec{Y}, 0)] \quad (5.5)$$

The integral on $d^3\vec{Y}$ is again to be evaluated by stationary phase. The stationary phase path is evidently the unperturbed ray joining the origin to the observation point \vec{x} . Hence, we may write

$$\langle |X(\vec{x})|^2 \rangle = \left(\frac{1}{4\pi k} \right)^2 \int_0^{\vec{x}} ds \int d^2\vec{q}_\perp(s) \tilde{C}(\vec{q}_\perp(s), \vec{Y}(s)) \quad (5.6)$$

in complete parallel to the homogeneous case. Here the line integral on ds is along the unperturbed ray, $\vec{q}_\perp(s)$ refers to the component of \vec{q} perpendicular to the ray at s , $\vec{Y}(s)$ is a point on the ray at s , and

$$\tilde{C}(\vec{q}, \vec{Y}) \equiv \int d^3\vec{y} e^{i\vec{q} \cdot \vec{y}} C(\vec{y}, \vec{Y}) \quad (5.7)$$

Next we turn to $\langle X^2 \rangle$:

$$\begin{aligned}
\langle X(\vec{x})^2 \rangle = & \int d^3\vec{Y} \frac{K(\vec{x},\vec{Y})K(\vec{Y},0)}{K(\vec{x},0)} \int d^3\vec{y} C(\vec{y},\vec{Y}) \\
& \exp i k \left[S(\vec{x},\vec{Y} + \vec{y}/2) + S(\vec{x},\vec{Y} - \vec{y}/2) \right. \\
& + S(\vec{Y} + \vec{y}/2,0) + S(\vec{Y} - \vec{y}/2,0) \\
& \left. - 2 S(\vec{x},0) \right] .
\end{aligned} \tag{5.8}$$

Now when we expand the exponent in powers of \vec{y} the linear terms vanish, so that we have

$$\begin{aligned}
\langle X(\vec{x})^2 \rangle = & \int d^3\vec{Y} \frac{K(\vec{x},\vec{Y})K(\vec{Y},0)}{K(\vec{x},0)} e^{2ik(S(\vec{x},\vec{Y})+S(\vec{Y},0)-S(\vec{x},0))} \\
& \int d^3\vec{y} C(\vec{y},\vec{Y}) \exp ik/4 y_i y_j A_{ij}(\vec{Y})
\end{aligned} \tag{5.9}$$

where we define

$$A_{ij}(\vec{Y}) = \frac{\partial}{\partial Y_i} \frac{\partial}{\partial Y_j} \left[S(\vec{x},\vec{Y}) + S(\vec{Y},0) \right] . \tag{5.10}$$

Evaluation of the integral on $d^3\vec{Y}$ by stationary phase again selects as the stationary phase path the unperturbed ray joining 0 to \vec{x} . The integral on $d^3\vec{y}$ can then be done by introducing the Fourier transform $\tilde{C}(\vec{q},\vec{Y})$ as in Eq. (5.7). Finally, we obtain

$$\langle X(\vec{x})^2 \rangle = - \left(\frac{1}{4\pi k} \right)^2 \int_0^{\vec{x}} ds \int d^2 q_{\perp}(s) \tilde{C}(\vec{q}_{\perp}(s), \vec{Y}(s)) e^{\frac{i}{k} q_{\perp i}(s) q_{\perp j}(s) A^{-1}(\vec{Y}(s))_{ij}} \quad (5.11)$$

The notation is as in Eq. (5.6), and the result is again in complete analogy to the homogeneous case.

Most of the comments we made in Section III concerning the results in the homogeneous background apply here as well. The expression for $\langle |X|^2 \rangle$ is again just that obtained in the geometrical optics approximation, but that for $\langle X^2 \rangle$ is not. Geometrical optics for $\langle X^2 \rangle$ is valid provided that

$$\frac{i}{k} q_{\perp i}(s) q_{\perp j}(s) A^{-1}(\vec{Y}(s))_{ij} \ll 1 \quad (5.12)$$

This is the analogue of the Fresnel condition.

In the homogeneous case, this condition boiled down to the requirement that

$$x < k L^2 \quad .$$

In the presence of a sound channel, the restriction (5.12) on the range is less severe; the condition (5.12) reduces approximately to

$$\frac{x}{k L_H^2} + \frac{x \tan^2 \theta}{k L_V^2} \ll 1 \quad (5.13)$$

where $\tan \theta$ is the maximum ray inclination to the horizontal. Since L_H is much larger than L_V , and since $\tan \theta$ is small, this restriction is

easier to meet than $x/k L_V^2 \ll 1$. Thus, in the presence of a sound channel, geometrical optics should be valid to a greater range than would be the case with a uniform background sound speed.

VI CONCLUSION

We have presented relatively simple formulas for phase and amplitude fluctuations of pressure signals in the ocean in the presence of a specified spectrum of sound-speed fluctuations for both a homogeneous ocean and one with a sound channel. These formulas reduce to the geometrical optics approximation for short ranges, but for long ranges they give results far less divergent with range than does geometrical optics.

For the case of a sound channel, where there are in general many rays, our results apply to fluctuations in the contribution of a single ray to the received pressure; thus the approximation is limited to ranges at which rays are still separable. Fluctuations in the total received pressure are much greater, due to interference between different rays, and are insensitive to details of the fluctuations in a single ray.⁴

The next step to be undertaken is to use the results outlined here, together with a semi-empirical fluctuation spectrum, to make numerical estimates of the fluctuations for the purpose of comparison with experiments. This will be described elsewhere.

REFERENCES

1. C. G. Callan and F. Zachariasen, "Low-Frequency Sound Propagation in a Fluctuating Infinite Ocean," SRI Technical Report JSR-73-10, Contract No. DAHC15-73-C-0370, SRI Project 3000, Menlo Park, Calif., (April 1974).
2. L. A. Chernov, Wave Propagation in a Random Medium (McGraw-Hill Book Co., New York, N.Y., 1960).
3. W. H. Munk, "Sound Channel in an Exponentially Stratified Ocean, with Application to SOFAR," J. Acoust. Soc. Am. Vol. 55, p. 220, (February 1974).
4. F. J. Dyson and W. H. Munk, J. Acoust. Soc. Am., to be published.

DISTRIBUTION LIST

Dr. Henry D. I. Abarbanel
National Accelerator Laboratory
P.O. Box 500
Batavia, Illinois 60510

Aerospace Corporation
Attn: Dr. Thomas Taylor
P.O. Box 5866
San Bernardino, California 92408

Dr. A. L. Anderson
Applied Research Laboratory
University of Texas
P.O. Drawer 8029
Austin, Texas 78712

Dr. V. C. Anderson
Scripps Institution of Oceanography
University of California
La Jolla, California 92037

Applied Research Laboratory
Pennsylvania State University
Attn: W. Baker
H. M. Jensen
S. McDaniel
Dr. H. S. Piper
D. C. Stickler
P.O. Box 30
State Collage, Pennsylvania 16801

Capt. J. C. Bajus Code 03
Naval Electronic Systems Command Hdqs.
Department of the Navy
Washington, D.C. 20360

Dr. R. W. Bannister
Defense Scientific Establishment
HMNZ Dockyard, Devonport
Auckland 45400, New Zealand

Bell Telephone Laboratories
Attn: R. Hardin
R. Holford
T. E. Talpey
Whippany, New Jersey 07901

Dr. H. F. Bezdek
Office of Naval Research (480)
Department of the Navy
Arlington, Virginia 22217

Prof. Theodore G. Birdsall, Director
Cooley Electronics Laboratory
Cooley Bldg., North Campus
University of Michigan
Ann Arbor, Michigan 48105

Dr. Martin H. Bloom
Polytechnic Institute of Brooklyn
Route 110, Room 205
Farmingdale, New York 11735

Cmdr. A. G. Brookes, Jr.
Office of Naval Research (Code 102-OS)
Department of the Navy
Arlington, Virginia 22217

Brown University
Department of Mathematics
Attn: C. Dafermos
W. Strauss
Providence, Rhode Island 02912

Mr. D. G. Browning
New London Laboratory
Naval Underwater Systems Center
New London, Connecticut 06320

Mr. B. M. Buck
Polar Research Laboratory, Inc.
123 Santa Barbara St.
Santa Barbara, California 93101

California Institute of Technology
Attn: Dr. Milton Plesset
Dr. Toshi Kubota
1201 East California Blvd.
Pasadena, California 91109

Dr. Curtis G. Callan, Jr.
Department of Physics
Princeton University
Princeton, New Jersey 08540

Dr. G. F. Carrier
Harvard University
Pierce Hall
Cambridge, Massachusetts 02139

Catholic University
Attn: F. Andrews
H. Uberall
620 Michigan Ave., NE
Washington, D.C. 20017

Courant Institute

Attn: D. Ahluwalia

C. Aquista

J. Bazer

R. Burridge

J. Keller

C. Morawetz

G. Papanicolaou

P. Sulem

J. Weileman

251 Mercer St.

New York, New York 10012

Capt. C. G. Darrell

Director, Undersea Warfare Technology

Office of Naval Research (Code 412)

Arlington, Virginia 22217

Dr. Roger F. Dashen

Institute for Advanced Study

Princeton, New Jersey 08540

Defence Research Establishment
Atlantic

Attn: Mr. J. B. Franklin

Dr. R. S. Thomas

P.O. Box 1012

Dartmouth, Nova Scotia, Canada

Defence Research Establishment
Pacific

Department of National Defence

Attn: Dr. R. P. Chapman

Mr. J. M. Thorliefson

Forces Mail Office

Victoria, B.C., VOS 1B0, Canada

Defense Advanced Research Projects
Agency

Attn: Mr. Robert M. Chapman, T10

Mr. Craig W. Hartsell, Jr., STO

Dr. George H. Heilmeyer, Director

Dr. Richard F. Hogle, T10

Dr. Donald J. Looft, Dep. Director

Mr. Robert A. Moore, T10

Dr. Stephen Zakany

1400 Wilson Boulevard

Arlington, Virginia 22209

Prof. I. Dyer

MIT

Department of Ocean Engineering

Cambridge, Massachusetts 02139

Prof. Freeman J. Dyson

Institute for Advanced Study

Princeton, New Jersey 08540

Dr. Harlow Farmer

P.O. 1925 (Main Station)

Washington, D.C. 20013

Dr. F. Fisher

Scripps Institution of Oceanography

University of California, San Diego

La Jolla, California 92037

Dr. R. M. Fitzgerald

Naval Research Laboratory

Department of the Navy

Washington, D.C. 20375

Dr. Stanley M. Flatté

360 Moore St.

Santa Cruz, California 95064

Flow Research, Inc.

Attn: Dr. Denny R. S. Ko

Dr. Michael Y. H. Pao

1810 South Central Ave.

Suite 72

Kent, Washington 98031

General Research Corp.

Attn: Mr. P. Donohoe

1501 Wilson Blvd.

Suite 700

Arlington, Virginia 22209

George Washington University

Computer Science and E.E.

Attn: R. Lang

Washington, D.C. 20006

Dr. R. R. Goodman Code 8000

Associate Director of Research

Naval Research Laboratory

Department of the Navy

Washington, D.C. 20375

Mr. W. Griswold

U.S. Naval Ship Research and

Development Center

Annapolis, Maryland 21402

Mr. I. Hagan

Royal Australian Navy Research

Laboratory

New Beach Road

Edgecliff, NSW, Australia 2027

Mr. G. R. Hamilton, Director
Ocean Science and Technology Division
Office of Naval Research
Department of the Navy
Arlington, Virginia 22217

Dr. J. S. Hanna
Office of Naval Research (AESD)
Department of the Navy
Arlington, Virginia 22217

Hawaii Institute of Geophysics
University of Hawaii
Attn: Dr. W. E. Hardy
Mr. P. M. Volk
2525 Correa Rd.
Honolulu, Hawaii 96822

Dr. J. B. Hersey
Bldg. 58
Naval Research Laboratory
Washington, D.C. 20390

Dr. C. W. Horton, Sr.
Applied Research Laboratory
University of Texas
P.O. Drawer 8029
Austin, Texas 78712

Dr. Norden Hueng
Department of Geosciences
North Carolina State University
Raleigh, North Carolina 27607

Hydronautics, Inc.
Attn: Dr. O. M. Phillips
Dr. T. R. Sundaram
Dr. J. Wu
Pindell School Rd., Howard Co.
Laurel, Maryland 20810

Imperial College
Department of E.E.
Attn: R. Clarke
Exhibition Road
London SW7, England

Institute for Acoustical Research
University of Miami
Attn: Dr. J. G. Clark
Dr. Martin Kronengold, Director
615 SW Second Ave.
Miami, Florida 33130

Institute for Defense Analysis
Attn: Dr. Joel Bengston
Mr. J. C. Nolen
Dr. Philip A. Selwyn
400 Army-Navy Drive
Arlington, Virginia 22202

Frank Jablonski (OPNAV)
Room 4B489
The Pentagon
Washington, D.C. 20350

Capt. D. M. Jackson
Naval Electronic Systems
Command Hdqs.
PME-124
Washington, D.C. 20360

Johns Hopkins University
Applied Physics Laboratory
Attn: L. Cronvich
H. Gilreath
N. Nicholas
A. Stone
8621 Georgia Ave.
Silver Spring, Maryland 20910

Dr. I.S.F. Jones
Head, Ocean Science Group
Royal Australian Navy Research
Laboratory
New Beach Road
Edgecliff, NSW, Australia 2027

Lamont-Doherty Geological Observatory
Columbia University
Attn: Mr. J. I. Ewing
Dr. H. Kutshale
Palisades, New York 10964

Dr. Frank Lane
KLD Associates, Inc.
7 High St., Suite 204
Huntington, New York 11743

Dr. W. S. Lewellen
Aeronautical Research Associates
of Princeton, Inc.
50 Washington Rd.
Princeton, New Jersey 08540

Dr. Harold W. Lewis
Department of Physics
University of California
Santa Barbara, California 93106

Mr. P. H. Lindop
Admiralty Research Laboratory
Teddington, Middlesex
England

Dr. S. W. Marshall
Naval Research Laboratory
Department of the Navy
Washington, D.C. 20375

Massachusetts Institute of Technology
Department of Mechanical Engineering
Attn: Dr. Eric L. Mollo-Christenson
Cambridge, Massachusetts 02139

Massachusetts Institute of Technology
Lincoln Laboratory
Attn: Dr. Charles W. Rook
P.O. Box 73
Lexington, Massachusetts 02173

Prof. H. Medwin
Naval Postgraduate School
Department of Physics
Monterey, California 93940

Cmdr. R. Miller (OPNAV)
Room 5D572
The Pentagon
Washington, D.C. 20390

Dr. G. A. Morgan
Embassy of Australia
1601 Massachusetts Ave., NW
Washington, D.C. 20036

Dr. W. Moseley Code 8160
Naval Research Laboratory
Department of the Navy
Washington, D.C. 20375

Dr. Walter H. Munk
9530 La Jolla Shores Drive
La Jolla, California 92037

NATO
SACLANT ASW Research Center
Attn: Dr. Bruce Williams
Dr. D. Wood
Viale San Bartolomeo, 400
I-19026 La Spezia, Italy

Naval Air Development Center
Attn: C. Bartberger
E. P. Garabed Code 2052
Warminster, Pennsylvania 18974

Naval Air Systems Command
Main Navy Bldg.
Attn: I. H. Gatzke
Washington, D.C. 20360

Naval Oceanographic Office
Attn: Mr. W. H. Geddes
Dr. M. Schulkin
Suitland, Maryland 20373

Naval Ordnance Laboratory
White Oak
Attn: I. Blatstein
Ms. E. A. Christian
R. J. Urick
Silver Spring, Maryland 20910

Naval Research Laboratory
Attn: R. Baer F. Ingenito
A. Berman Frank MacDonald
J. Cybulsky J. McCoy
J. Dugan S. Piacsek
J. O. Elliot K. G. Williams
B. Hurdle
4555 Overlook Ave., SW
Washington, D.C. 20390

Naval Scientific and Technical
Intelligence Center
Attn: Capt. J. P. Prisley
4301 Suitland Rd.
Washington, D.C. 20390

Naval Ship Systems Command
TRIDENT Project Officer
Washington, D.C. 20360

Naval Undersea Center
Attn: H. S. Aurand, Jr., Head,
Ocean Acoustics Division
Dr. H. Bucker J. Neubert
Dr. A. G. Fabula M. A. Pedersen
E. Floyd C. Ramstedt
D. Gordon D. White
H. Morris
San Diego, California 92132

Naval Undersea Systems Center
Attn: Dr. J. D'Albora
Newport, Rhode Island 02840

Naval Underwater Systems Center
New London Laboratory

Attn: D. Abraham A. Krulisch
 D. G. Browning R. Lauer
 J. Cohen G. Leibiger
 H. Cox R. L. Martin
 R. Deavenport R. H. Mellen
 F. Dinapoli J. Papadakis
 L. Einstein W. I. Roderick
 A. Ellinthorpe R. Saenger
 R. Elswick P. Scully-Power
 H. Freese D. Shonting
 O. D. Grace L. Stallworth
 J. Hassab W. A. Von Winkle
 E. Jensen H. Weinberg
 W. Kanabis

New London, Connecticut 06320

Dr. William A. Nierenberg
Scripps Institution of Oceanography
University of California
La Jolla, California 92037

ODDR&E

The Pentagon

Attn: G. Cann, Rm. 3D1048
 Dr. D. R. Heebner, Rm 3E1040
 N. F. Wikner, Rm 3E1087
Washington, D.C. 20301

Office of Naval Research

Attn: Dr. W. J. Condell
 Mr. A. O. Sykes, Code 412
800 N. Quincy St.
Arlington, Virginia 22217

Office of Naval Research

Attn: D. Cacchione
495 Summer St.
Boston, Massachusetts 22210

Office of Naval Research
Acoustic Environmental Support
Detachment

Attn: R. Buchal
 Dr. R. C. Cavanagh
 A. Cecelski
 S. Reed, Jr.
 C. W. Spofford
 Lcdr. B. T. Steele
 Cdr. P. R. Tatro, Head
Arlington, Virginia 22217

Office of Naval Research

Department of the Navy

Attn: Cdr. J. Ballou
 M. Cooper
 R. Cooper
 J. Witting

Washington, D.C. 20360

Physical Dynamics, Inc.

Attn: Dr. Alex Thomson
 Dr. Bruce West

P.O. Box 1069
Berkeley, California 94704

Capt. Daniel Piraino (NSP)
SP202

Department of the Navy
Washington, D.C. 20390

Polytechnic Institute of New York

Department of Mathematics

Attn: H. Hockstadt
333 Jay St.
Brooklyn, New York 11201

James H. Probus (OASN)

Room 4E741

The Pentagon

Washington, D.C. 20350

R&D Associates

Attn: Dr. F. L. Fernandez
 Dr. D. Holliday
 Dr. M. Milder

P.O. Box 3580
Santa Monica, California 90403

Dr. Gordon Raisbeck

Arthur D. Kittle, Inc.

Cambridge, Massachusetts 02140

Rensselaer Polytechnic Institute

Department of Mathematics

Attn: M. Jacobson
Troy, New York 12181

Riverside Research Institute

Attn: Dr. Marvin King
80 West End Ave.
New York, New York 10023

Mr. D. A. Rogers (NSP)

SP2018

Department of the Navy
Washington, D.C. 20390

Mr. Benjamin Rosenberg (OPNAV)
Room 4B513
The Pentagon
Washington, D.C. 20350

Rosentiel School of Marine and
Atmospheric Science
University of Miami
Attn: Dr. S. C. Daubin
Dr. H. DeFarrari
Miami, Florida 33149

Dr. Donald Ross
Tetra Tech, Inc.
3559 Kenyon Street
San Diego, California 92110

Science Application, Inc.
Attn: H. Wilson
P.O. Box 351
La Jolla, California 92037

Mr. Carey Smith
Naval Sea Systems Command Hdqs.
Code 06H1
Department of the Navy
Washington, D.C. 20360

Stanford Research Institute
Attn: N. Cianos, 44
H. Guthart, 404B
R. C. Honey, 406A
K. Krishnan, K1068
333 Ravenswood Ave.
Menlo Park, California 94025

TRW Systems Group
Attn: E. Baum
J. Chang
One Space Park
Redondo Beach, California 90278

Dr. F. Tappert
Courant Institute
251 Mercer St.
New York, New York 10012

Dr. C. M. Tchen
1380 Riverside Drive
New York, New York 10033

University of California
Lawrence Radiation Laboratory
Attn: Dr. Harold P. Smith
Dr. M. Wirth
P.O. Box 808
Livermore, California 94550

University of California, San Diego
Attn: Dr. C. Cox
Dr. R. Davis
Dr. C. Gibson
Dr. J. Miles
P.O. Box 119
La Jolla, California 92038

University of California
Scripps Institution of Oceanography
Marine Physical Laboratory
Attn: Dr. T. Foster
Dr. G. B. Morris
Dr. F. Spiess
San Diego, California 92152

University of Rhode Island
Attn: G. Ladas E. Roxin
L. LeBlanc R. Suryanarayan
F. Middleton G. Verma
A. Poularicas
Kingston, Rhode Island 02881

Dr. Roberto Vaglio-Laurin
Advanced Technology Laboratories, Inc.
400 Jericho Turnpike
Jericho, New York 11753

Virginia Polytechnic Institute
State University
Attn: I. Bezieris
W. Kohler
Blacksburg, Virginia 24061

Cdr. Don Walsh (OASN/R&D)
Room 4E741
The Pentagon
Washington, D.C. 20350

Dr. Kenneth M. Watson
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

Wayne State University
Department of Mathematics
Attn: P. Chow
Detroit, Michigan 46202

Dr. M. S. Weinstein
Underwater Systems, Inc.
8121 Georgia Ave., Suite 700
Silver Spring, Maryland 20910

Western Electric Co.
Bell Telephone Laboratories
Attn: Dr. E. Y. Harper
Dr. F. Labianca
Whippany, New Jersey 07981

M. Wilson
Naval Ship Research and Development
Center
Washington, D.C. 20007

Woods Hole Oceanographic Institution
Attn: Dr. E. E. Hays
Dr. R. Porter
Dr. R. Spindel
Mr. W. A. Watkins
Woods Hole, Massachusetts 02543

Dr. J. L. Worzel
Marine Science Institute
Geophysics Laboratory
700 Thestrand
Galveston, Texas 77550

Dr. D. V. Wyllie
Weapons Research Establishment
Box 2151, G.P.O.
Adelaide, South Australia 5001

Dr. H. Yura
Aerospace Corporation
P.O. Box 92956
Los Angeles, California 90009

Dr. Fredrik Zachariasen
#452-48
Department of Physics
California Institute of Technology
Pasadena, California 91109